Multi-Agent Path Finding and Its Applications

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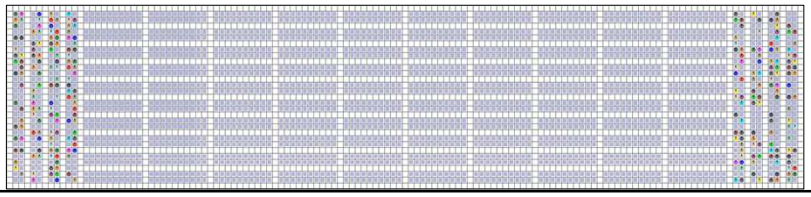
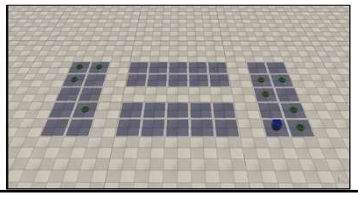
I will often say MAPF instead of multi-agent path finding.

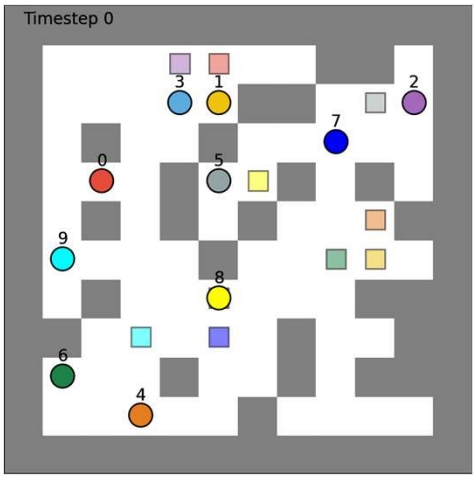




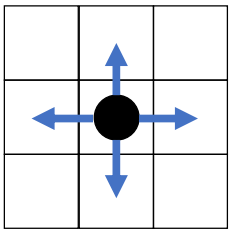
1

Multi-Agent Path Finding (MAPF)

- Optimization problem with the objective to minimize task-completion time (called makespan) or the sum of travel times (called flowtime)



Assumptions



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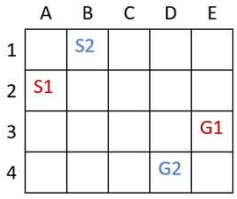
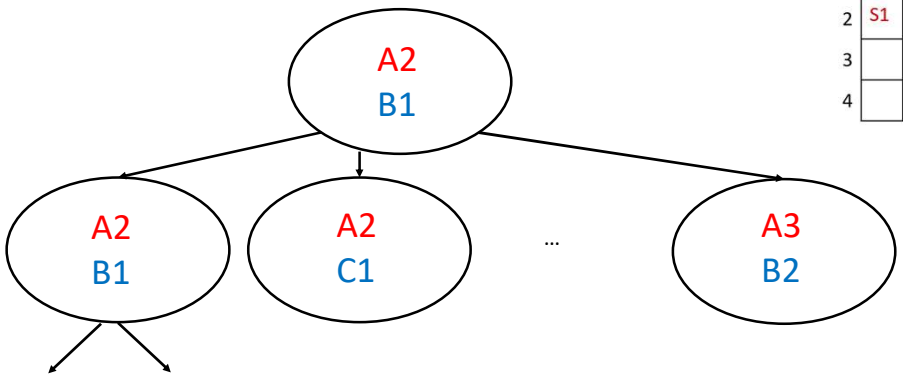
Multi-Agent Path Finding (MAPF)

not our work

- A*-based search in the joint state space



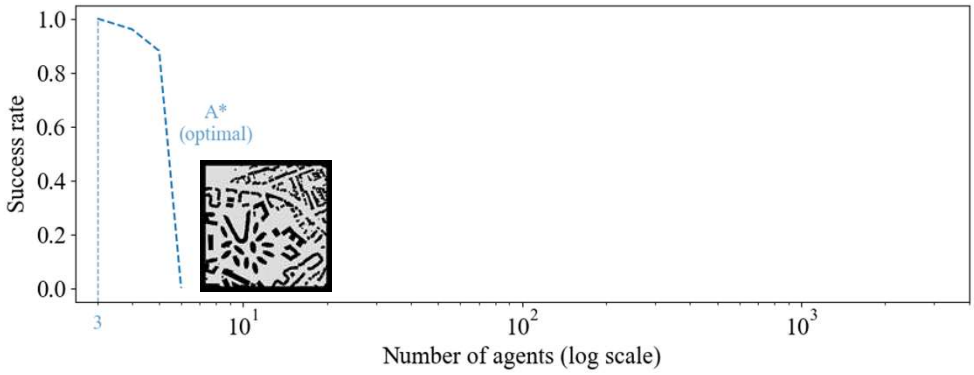


[1] T. Standley, "Finding Optimal Solutions to Cooperative Pathfinding Problems", AAAI, 2010.

6

Multi-Agent Path Finding (MAPF)

- Runtime limit: 1 minute



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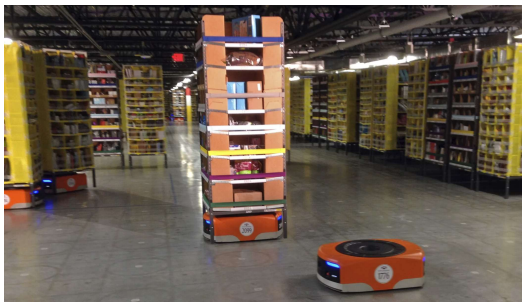
Automated Warehousing

not our work

- Amazon fulfillment centers
  - 2003 Kiva Systems founded
  - 2012 Amazon acquires Kiva Systems for $775 million
  - 2015 Kiva Systems becomes Amazon Robotics



Left photo: https://olhardigital.com.br/en/2019/11/25/cars-and-technology/amazon-wants-to-make-any-delivery-in-just-30-minutes/
Right photo: https://www.dailymail.co.uk/sciencetech/article-3229192/Would-orders-robot-Hitachi-hires-machine-manager-warehouse-workers.html

https://www.youtube.com/watch?v=YSrnV0wZywU

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Issues in Automated Warehousing

- We are only starting to study automated warehousing
  - When should one start to process a given order given that different orders have different delivery deadlines?
  - Where should one place a given shelf to maximize throughput, so that shelves that contain frequently ordered items can be fetched fast?
  - Which one of several possible shelves should one fetch to obtain an item for a given order?
  - Where should one place the corridors to maximize throughput? And how can one change the layout of a warehouse *during operation*?
  - How should one move the robots to avoid them obstructing each other and allow them to reach their goal locations quickly?

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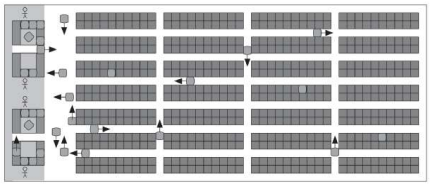
Automated Warehousing

not our work

- Amazon fulfillment centers – warehousing part

https://www.youtube.com/watch?v=8gy5tYVR-28



https://www.machinedesign.com/mechanical-motion-systems/article/21835788/changing-the-future-of-warehouses-with-amazon-robots

[1] P. Wurman et al., "Coordinating Hundreds of Cooperative, Autonomous Vehicles in Warehouses", AI Magazine 29(1), 9-20, 2008

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Automated Warehousing

not our work

- Amazon fulfillment centers – sorting part

https://www.wired.com/story/amazon-warehouse-robots/

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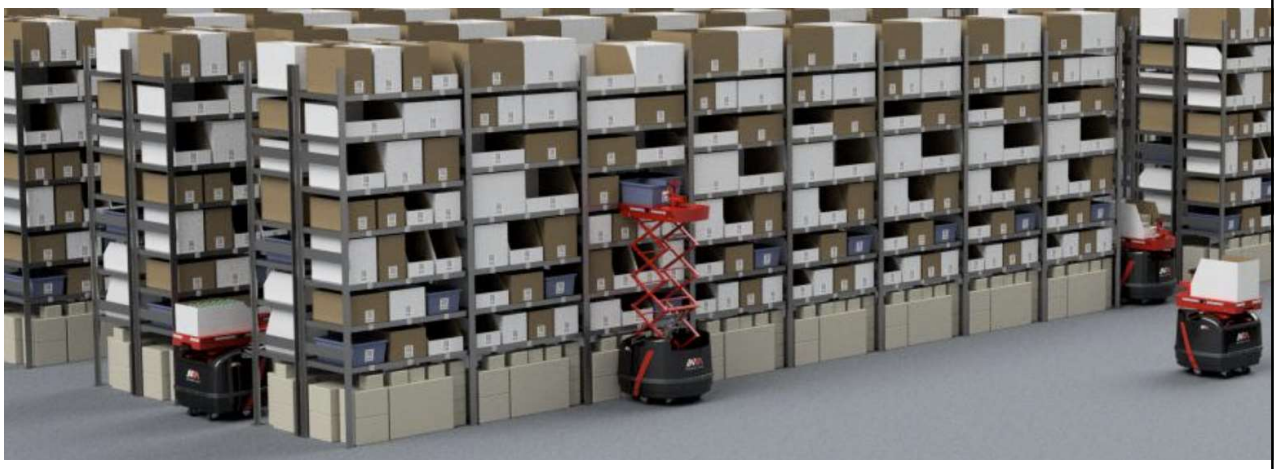
Automated Warehousing

not our work

- InVia Robotics



https://www.inviarobotics.com/app/uploads/2020/02/DASRS-11_0.jpg

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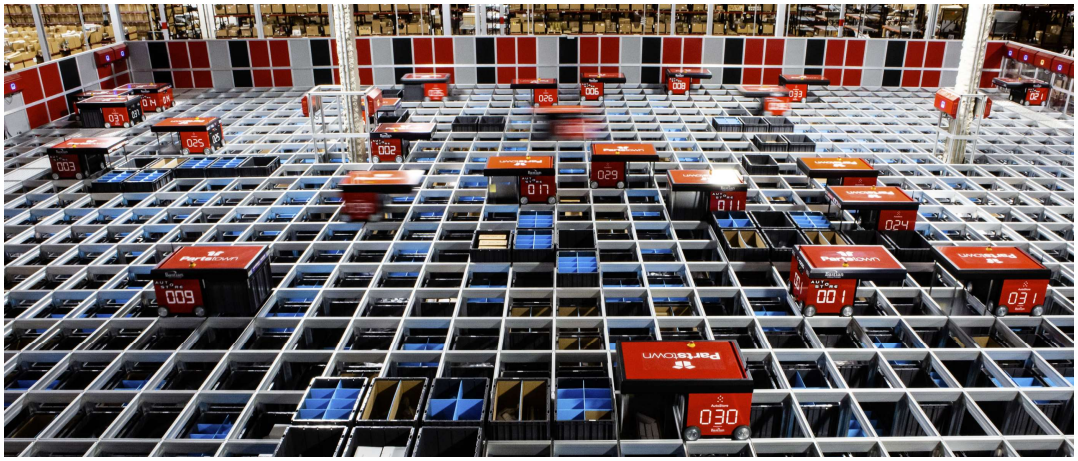
Automated Warehousing

not our work

- Autostore



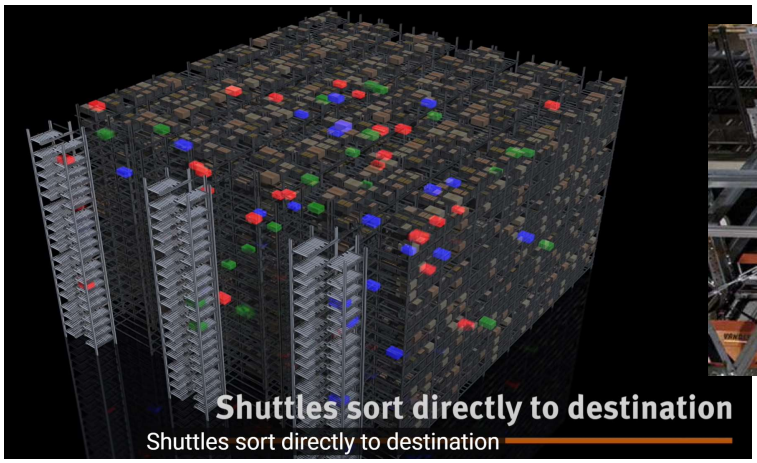
https://www.autostoresystem.com/cases/autostore-helps-parts-town-deliver-on-its-same-day-delivery-promise

23

Automated Warehousing

not our work

- Vanderlande ADAPTO



https://www.youtube.com/watch?v=Szt20xNxB5M

https://www.directindustry.com/prod/vanderlande/product-5796-1580097.html

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Manufacturing/Inspection

not our work

- B&R Automation ACOPOS 6D

https://www.br-automation.com/en/products/mechatronic-systems/acopos-6d/

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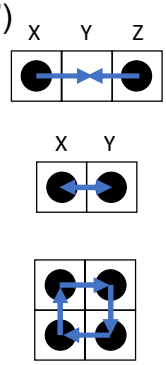
Multi-Agent Path Finding (MAPF)

- Assumptions
  - Each agent moves N, E, S or W to an adjacent unblocked cell or waits, in unit time
  - Not allowed ("vertex collision")
    - Agent 1 moves from X to Y
    - Agent 2 moves from Z to Y
  - Not allowed ("edge collision")
    - Agent 1 moves from X to Y
    - Agent 2 moves from Y to X
  - Allowed



[1] R. Stern et al., "Multi-Agent Pathfinding: Definitions, Variants, and Benchmarks", SoCS, 2019.

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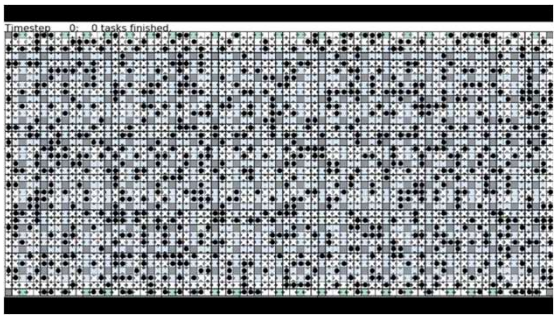
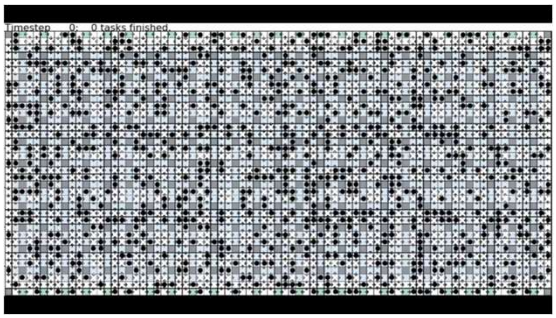
Multi-Agent Path Finding (MAPF)

- 800 robots (= 32% empty cells) on a 37x77 sorting-center map with 50 workstations and 275 chutes (joint project with Amazon Robotics)



Single-Agent Planner



Our MAPF Planner

https://www.wired.com/story/amazon-warehouse-robots/

[1] J. Li et al., "Lifelong Multi-Agent Path Finding in Large-Scale Warehouses", AAAI, 2021.

31

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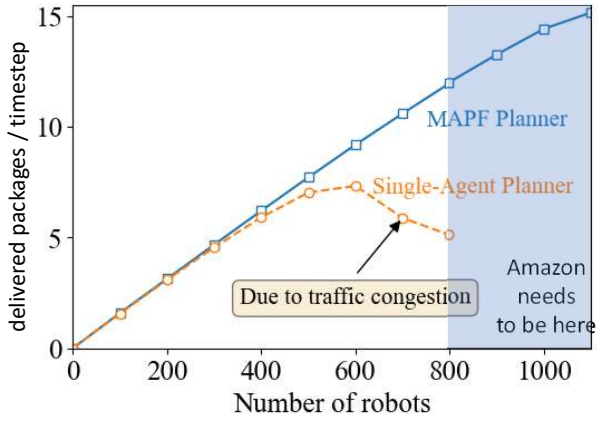

The Machine
Making sense of AI

Amazon's AI tool can plan collision-free paths for 1,000 warehouse robots

In a recent technical paper, researchers affiliated with the University of Southern California and Amazon Robotics explored a solution to the problem of lifelong multi-agent path finding (MAPF), where a team of agents must be moved

https://venturebeat.com/2020/05/18/amazons-ai-tool-can-plan-collision-free-paths-for-1000-warehouse-r

[1] J. Li et al., "Lifelong Multi-Agent Path Finding in Large-Scale Warehouses", AAAI, 2021.

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Main Talk

Achieving Practicality: Efficiency

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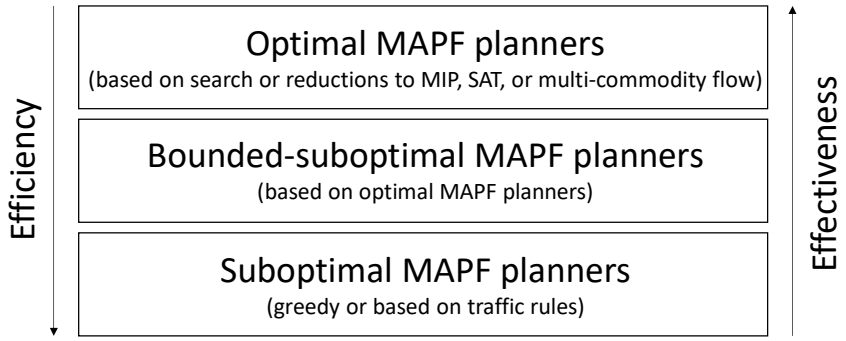
Multi-Agent Path Finding (MAPF)



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Complexity of MAPF

- Optimal or bounded-suboptimal MAPF planning
  - NP hard to find a makespan- or flow-time optimal MAPF plan [1], even on planar [2] or grid-like graphs [3]
  - Polynomal time to find a makespan-optimal MAPF plan with anonymous agents [4]
  - NP-hard to find a makespan-bounded-suboptimal MAPF plan with suboptimality factors of less than 4/3 [5]

https://www.pngegg.com/en/png-horua

[1] J. Yu and S. LaValle, "Structure and Intractability of Optimal Multi-Robot Path Planning on Graphs", AAAI, 2013.
[2] J. Yu, "Intractability of Optimal Multi-Robot Path Planning on Planar Graphs", IEEE Robotics and Automation Letters, 2016.
[3] J. Banfi et al., "Intractability of Time-Optimal Multi-Robot Path Planning on 2D Grid Graphs with Holes", IEEE Robotics and Auto
[4] J. Yu and S. LaValle, "Multi-Agent Path Planning and Network Flow", WAFR, 2012.
[5] H. Ma et al., "Multi-Agent Path Finding with Payload Transfers and the Package-Exchange Robot-Routing Problem", AAAI, 2016.

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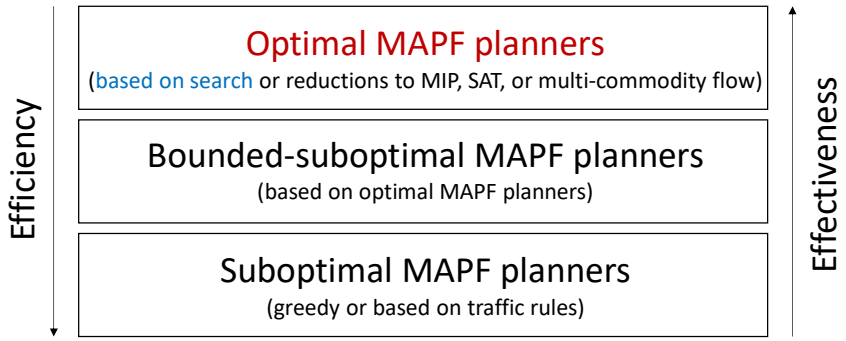
Multi-Agent Path Finding (MAPF)



The memory consumption is also important and can be decreased with iterative deepening, which enables the application of incremental search

[1] E. Boyarski et al., "Iterative-Deepening Conflict-Based Search", IJCAI 2020.

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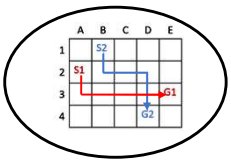
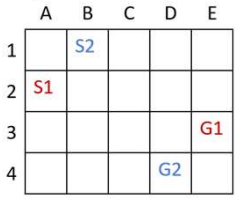
Conflict-Based Search (CBS)

not our work

- Conflict-based search (CBS) in the collision-resolution space





[1] G. Sharon et al., "Conflict-Based Search for Optimal Multi-Agent Pathfinding", Artificial Intelligence 219, 2015.

43

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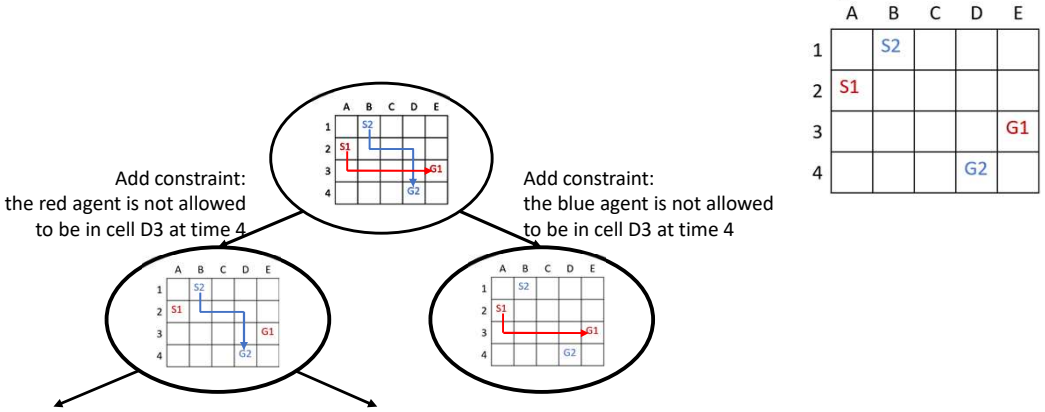

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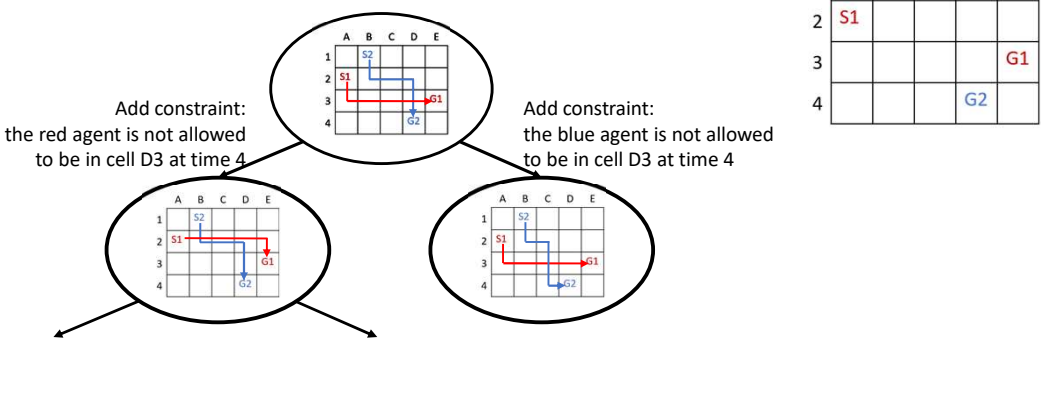

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45

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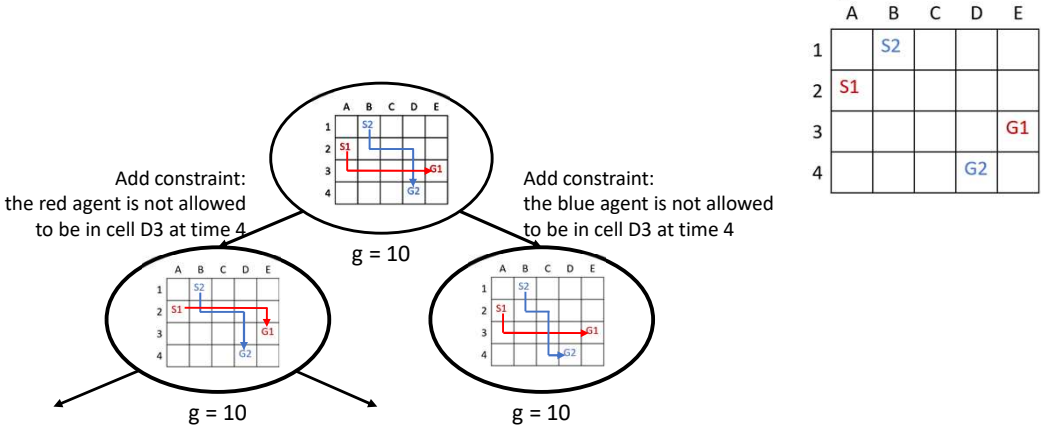

[1] G. Sharon et al., "Conflict-Based Search for Optimal Multi-Agent Pathfinding," Artificial Intelligence 219, 2015.

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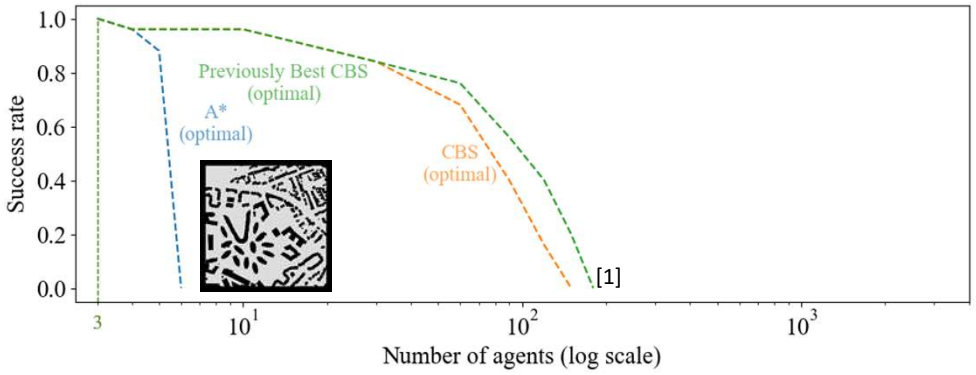
Multi-Agent Path Finding (MAPF)

- Runtime limit: 1 minute



[1] E. Boyarski et al., "ICBS: Improved Conflict-Based Search Algorithm for Multi-Agent Pathfinding", IJCAI, 2015.

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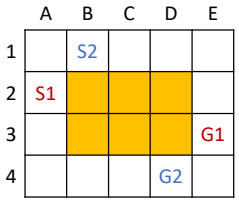
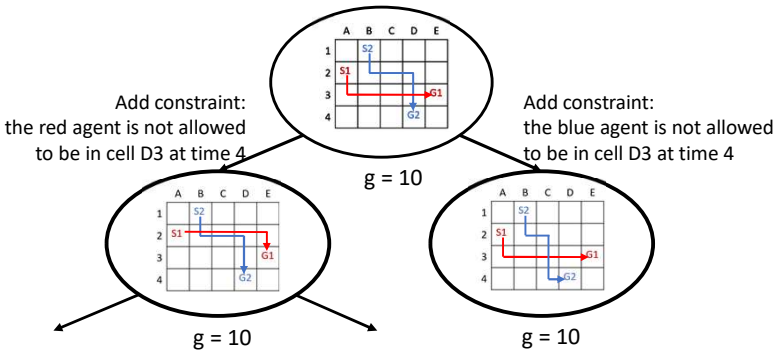
CBS Improvement 3: Symmetry Breaking

inspiration from constraint programming

- Conflict-based search (CBS) in the collision-resolution space





[1] J. Li et al., "Symmetry-Breaking Constraints for Grid-Based Multi-Agent Path Finding", AAAI, 2019.
[2] J. Li et al., "New Techniques for Pairwise Symmetry Breaking in Multi-Agent Path Finding", ICAPS, 2020.
[3] J. Li et al., "Pairwise Symmetry Reasoning for Multi-Agent Path Finding Search", Artificial Intelligence, 2021.

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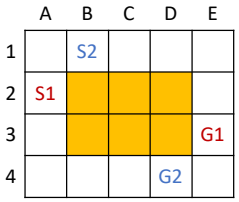
- Conflict-based search (CBS) in the collision-resolution space



	1	2	3	4	5	6	7	8	9
1	1	1	3	3	4	5	6	7	8
2		3	7	14	26	46	79	133	221
3			22	53	116	239	472	904	1,692
4				142	392	1,016	2,651	6,828	17,747
5					1,015	2,971	8,525	23,733	65,236
6						7,447	24,275	78,002	254,173
7							62,429	222,524	795,197
8								573,004	>1,518,151

[1] J. Li et al., "Symmetry-Breaking Constraints for Grid-Based Multi-Agent Path Finding", AAAI, 2019.
[2] J. Li et al., "New Techniques for Pairwise Symmetry Breaking in Multi-Agent Path Finding", ICAPS, 2020.

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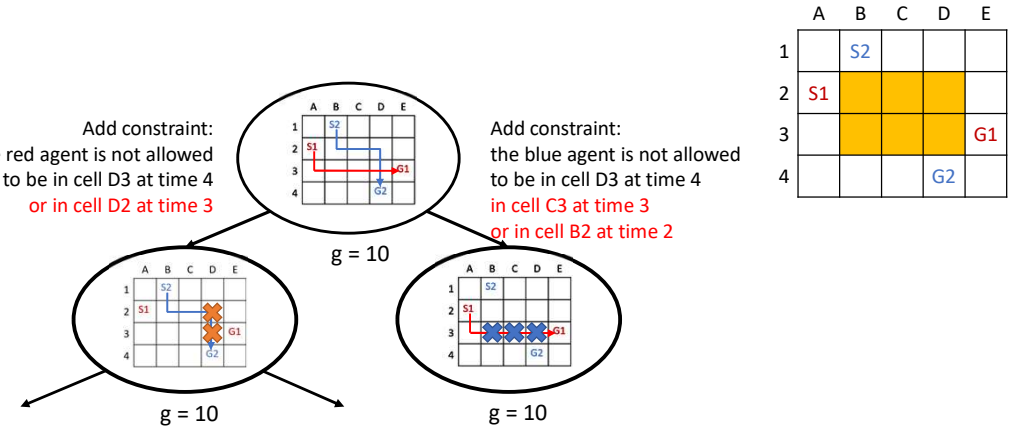

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CBS Improvement 3: Symmetry Breaking

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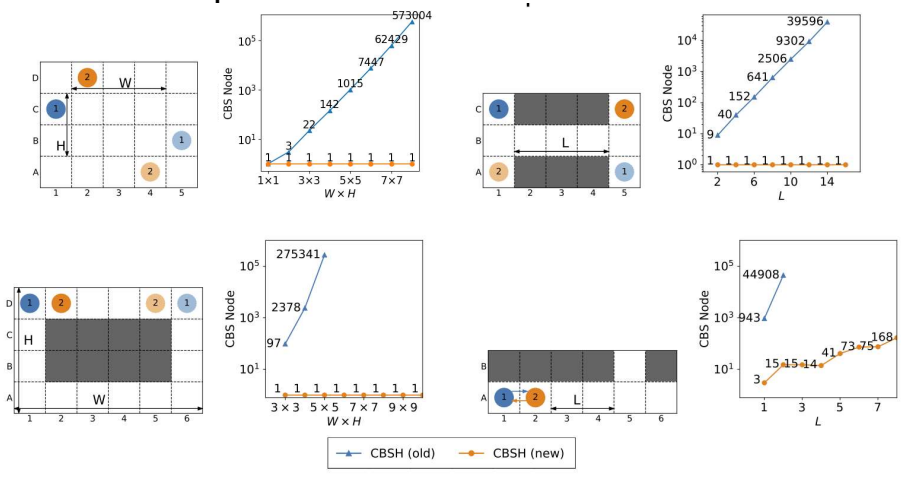
- Number of node expansions



[1] H. Zhang et al., "Multi-Agent Path Finding with Mutex Propagation", ICAPS, 2020.

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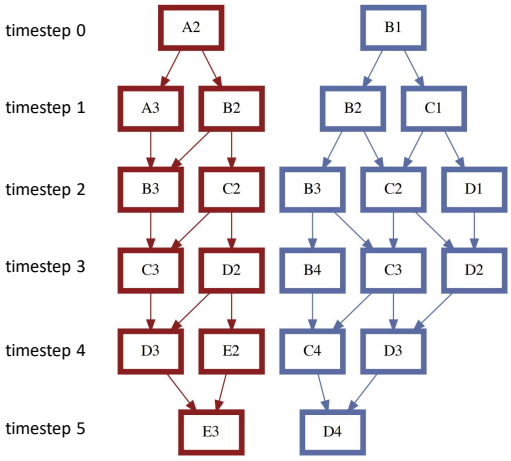
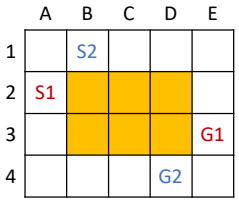
CBS Improvement 3: Symmetry Breaking

inspiration from symbolic planning

- Mutex propagation





[1] H. Zhang et al., "Multi-Agent Path Finding with Mutex Propagation", ICAPS, 2020.

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CBS Improvement 3: Symmetry Breaking

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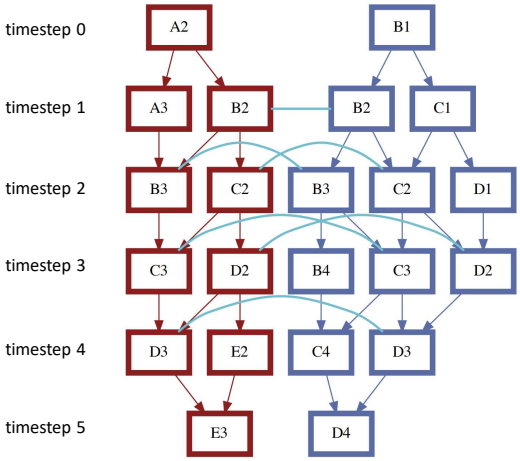

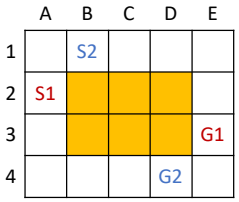

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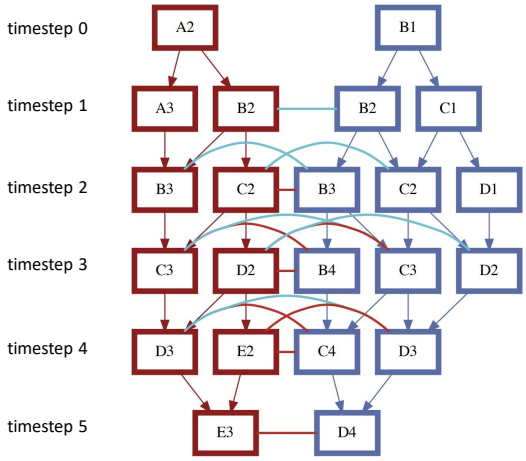

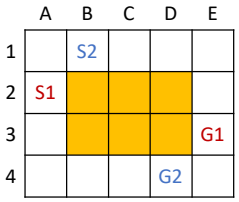

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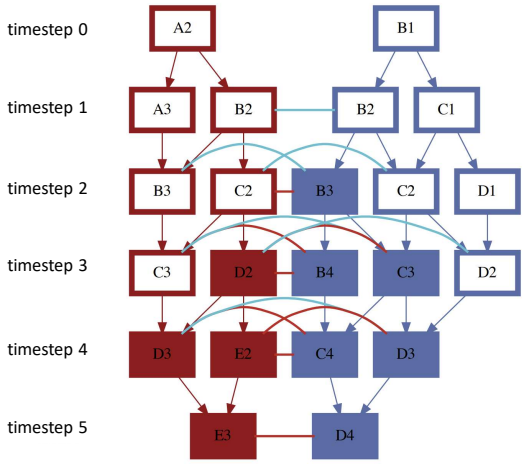

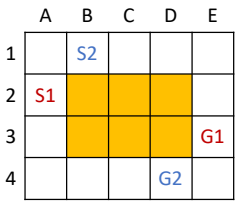

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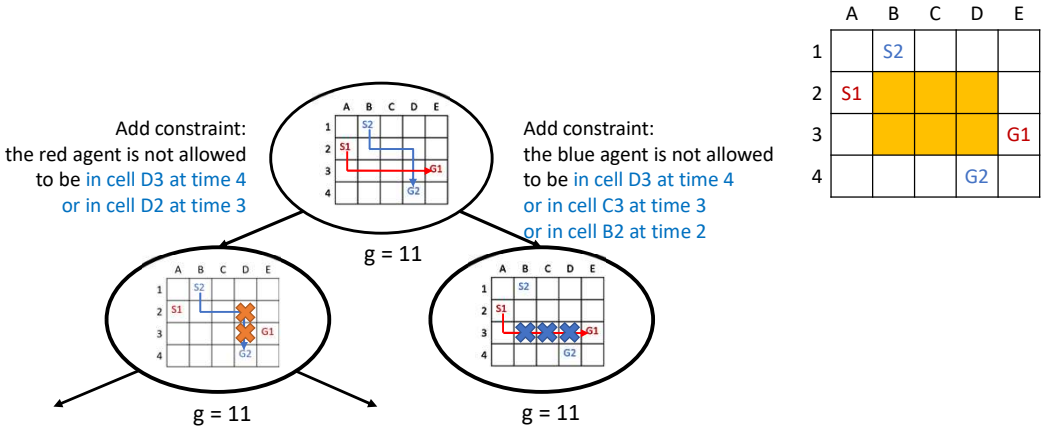
inspiration from symbolic planning

- Mutex propagation



[1] H. Zhang et al., "Multi-Agent Path Finding with Mutex Propagation", ICAPS, 2020.

65

Summary: Conflict-Based Search (CBS)

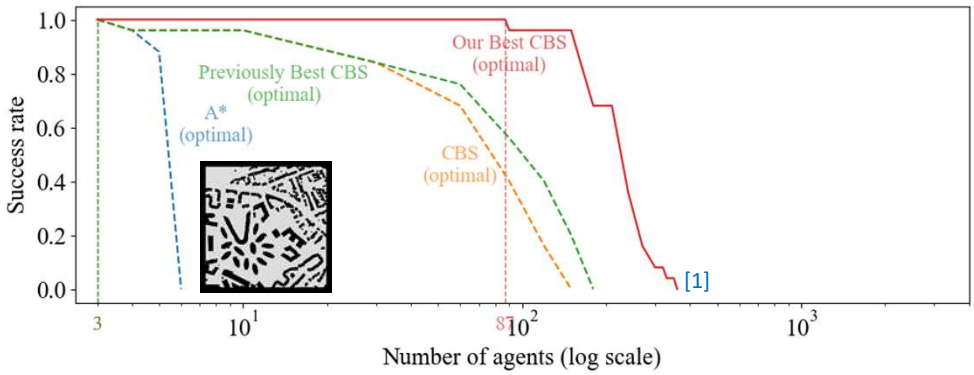
- Resolving collisions efficiently
  - Symmetry breaking with designed or discovered constraints (optimal)
  - Disjoint splitting (optimal)
  - Priority-based search (suboptimal)
- Selecting nodes cleverly
  - Heuristics (optimal)
  - Explicit Estimation Search (bounded-suboptimal)
- Reducing the number of collisions in a node
  - Highways (bounded-suboptimal)
  - Rolling-horizon collision resolution (suboptimal)
- Other techniques
  - Random restarts
  - Large neighborhood search with designed or learned neighborhoods

66

66

Conflict-Based Search (CBS)

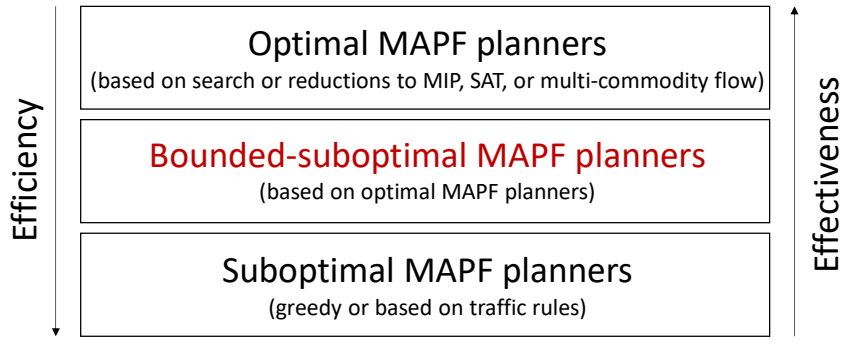
- Runtime limit: 1 minute



[1] J. Li et al., "Pairwise Symmetry Reasoning for Multi-Agent Path Finding Search", Artificial Intelligence, 2021.

69

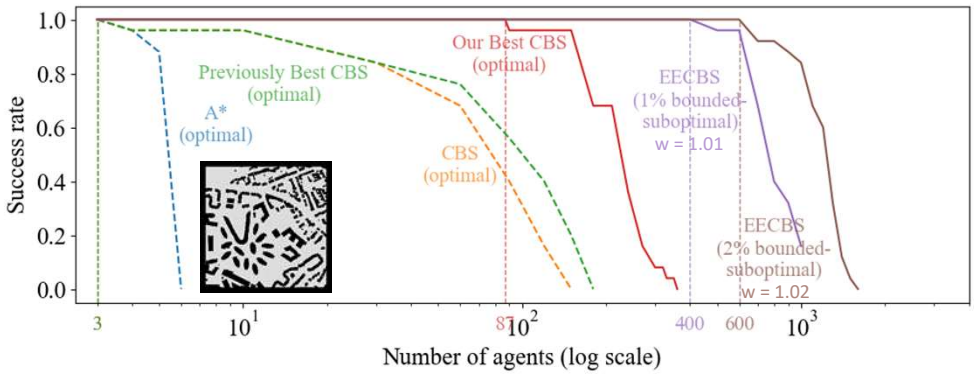
Multi-Agent Path Finding (MAPF)



70

Conflict-Based Search (CBS)

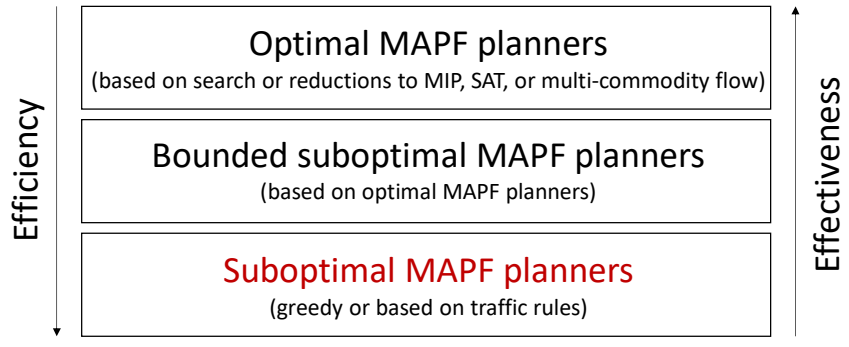
- Runtime limit: 1 minute



- Not included: directional lanes to avoid head-to-head collisions
- Not included: rapid random restarts

74

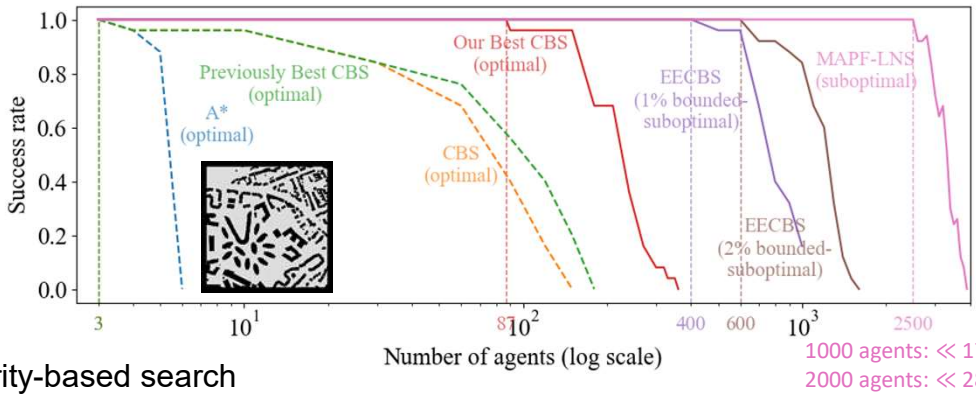
Multi-Agent Path Finding (MAPF)



88

Conflict-Based Search (CBS)

- Runtime limit: 1 minute



1000 agents: ≪ 17% suboptimal
2000 agents: ≪ 28% suboptimal
2500 agents: ≪ 32% suboptimal

- Priority-based search
- Large neighborhood search
- Not included: rolling-horizon collision resolution

[1] J. Li et al., "RMAPF: Fast Repairing for Multi-Agent Path Finding via Large Neighborhood Search", under review for AAAI, 2022.

89

Algorithm Selection

- Jingyao Ren and Nora Ayanian (USC)
- Roni Stern et al. (Ben-Gurion University)

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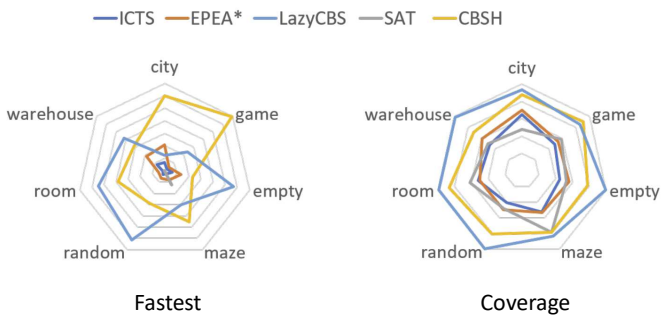
90

Algorithm Selection

not our work

Model	Fastest All	Fastest Even	Fastest Rand.	Coverage All	Coverage Even	Coverage Rand.
EPEA*	9.63	8.49	10.66	55.26	55.13	55.57
ICTS	4.23	2.73	5.21	49.60	48.99	50.32
CBS-H	**46.95**	**45.02**	**48.75**	83.20	66.01	**98.49**
MDD-SAT	2.11	2.48	1.72	57.92	57.48	58.54
Lazy CBS	37.26	41.28	33.66	**92.36**	**91.45**	93.17
Best-at-grid(C)	49.45	49.98	48.47	**97.62**	**97.43**	**98.55**
Best-at-grid(F)	**60.09**	**61.77**	**60.29**	92.46	91.53	97.80
Best-at-grid-type(C)	51.18	40.88	48.71	93.95	91.64	98.51
Best-at-grid-type(F)	57.47	58.00	58.11	92.57	89.81	98.48



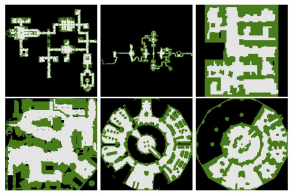
[1] O. Kaduri et al., "Experimental Evaluation of Classical Multi Agent Path Finding Algorithms", SoCS, 2021.

91

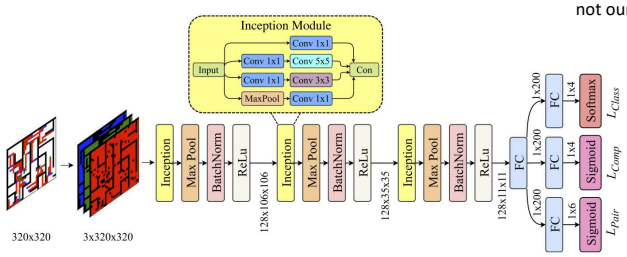
91

Algorithm Selection

not our work



Algorithm	Accuracy	Coverage	Runtime
CBS	0.1888	0.41	7,714
CBSH	0.2810	0.90	**2,211**
BCP	**0.5294**	**0.91**	2,256
SAT	0.0008	0.38	8,548
XGBoost Cl	0.6711	0.95	1,694
CNN_{Class}	0.7118	0.95	1,560
G2V	0.7130	0.95	1,548
MAPFAST	**0.7689**	**0.97**	**1,339**
Oracle	1.0	1.0	917



- *Accuracy* is the proportion of instances in which an algorithm is the fastest in the portfolio.
- *Coverage* is the proportion of the instances that an algorithm successfully solves within the time limit (5 minutes).
- *Runtime* is the overall time taken by the algorithm, in minutes, to solve all instances. A default value of 5 minutes is added to the runtime when the algorithm doesn't solve the input instance within time limit.

[1] J. Ren et al., "MAPFAST: A Deep Algorithm Selector for Multi Agent Path Finding", AAMAS, 2021.

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USC Viterbi School of Engineering

Main Talk

Achieving Practicality: Robustness

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USC Viterbi School of Engineering

CBS Extensions

- Agents of different kinodynamics
- Uncertainty about the speeds of agents

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Non-Holonomic Robots

- Application: loading docks

www.easternlifttruck.com

courtesy of Marcello Cirillo

Linde CiTi Truck at Örebro University

[1] M. Cirillo et al., "Integrated Motion Planning and Coordination for Industrial Vehicles", ICAPS 2014.

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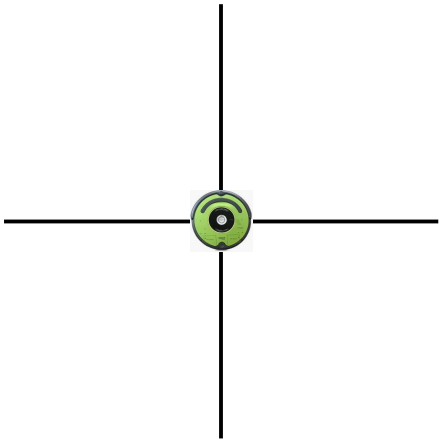
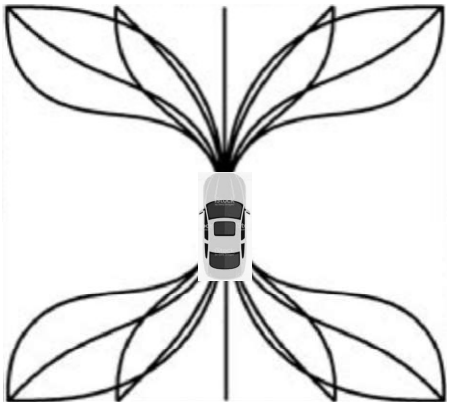
Non-Holonomic Robots



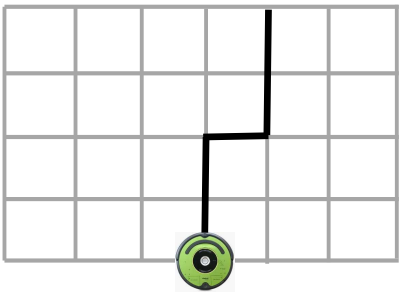


[1] L. Cohen et al., "Optimal and Bounded-Suboptimal Multi-Agent Motion Planning". SOCS 2019.
[2] M. Pivtoraiko et al., "Differentially Constrained Mobile Robot Motion Planning in State Lattices", Journal of Field Robotics 26(3),

106

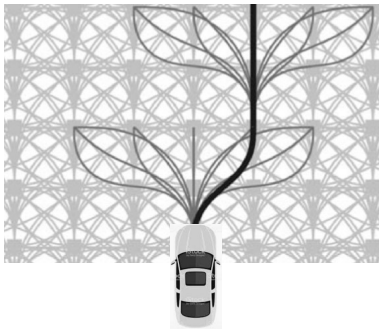
Non-Holonomic Robots

Grid



- Discretized space (x,y)
- Planar graph
- Uniform edge traversal times
- Discretized time

State Lattice

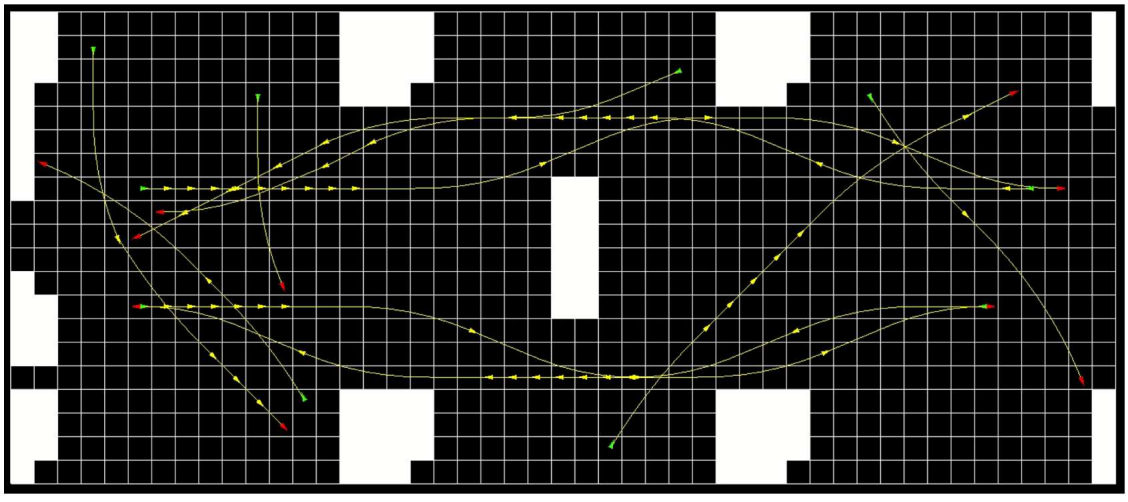


- Discretized space (x,y,orientation)
- Nonplanar graph
- Non-uniform edge traversal times
- Continuous time

[1] L. Cohen et al., "Optimal and Bounded-Suboptimal Multi-Agent Motion Planning". SOCS 2019.
[2] M. Pivtoraiko et al., "Differentially Constrained Mobile Robot Motion Planning in State Lattices", Journal of Field Robotics 26(3),

107

Non-Holonomic Robots



[1] L. Cohen et al., "Optimal and Bounded-Suboptimal Multi-Agent Motion Planning". SOCS 2019.
[2] M. Pivtoraiko et al., "Differentially Constrained Mobile Robot Motion Planning in State Lattices", Journal of Field Robotics 26(3),

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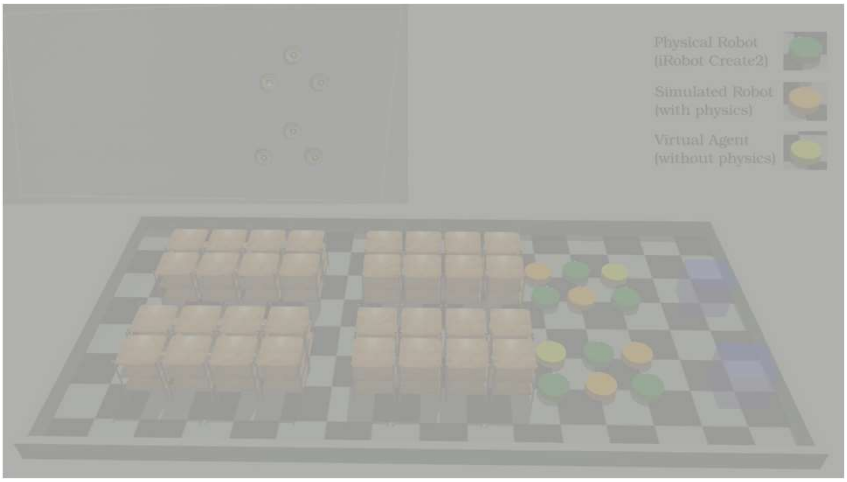
Robust Plan Execution

not our work

- Mixed reality simulation



[1] W. Hoenig et al., "Mixed Reality for Robots", IROS, 2015.

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Main Talk

A Challenge for the Audience

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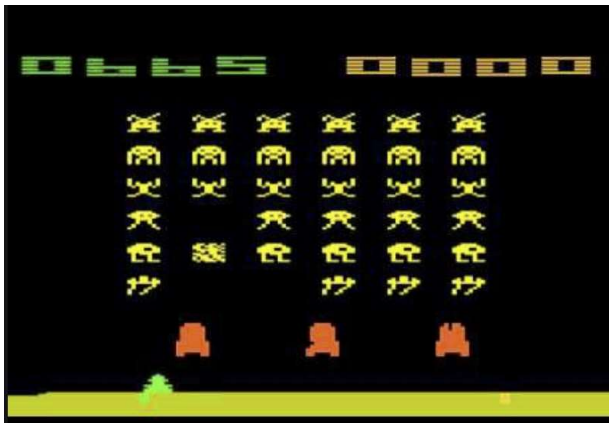
Machine Learning

not our work

- PRIMAL: mix of deep reinforcement learning and imitation learning



https://medium.com/free-code-camp/explained-simply-how-deepmind-taught-ai-to-play-video-games-9eb5f38c89ee https://www.pcworld.com/article/2889432/google-ai-program-masters-classic-atari-video-games.html

[1] V. Mnih et al., "Human Level Control through Deep Reinforcement Learning", Nature 518, 2015.

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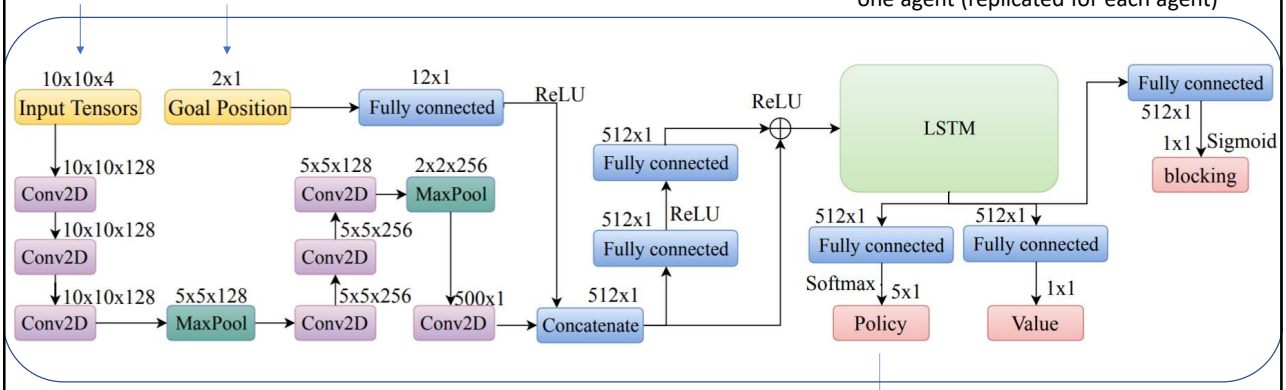
Machine Learning

- PRIMAL: mix of deep reinforcement learning and imitation learning

one agent (replicated for each agent)



Next action to execute

[1] G. Sartoretti et al., "PRIMAL: Pathfinding via Reinforcement and Imitation Multi-Agent Learning," IEEE RAL 4(3), 2019.

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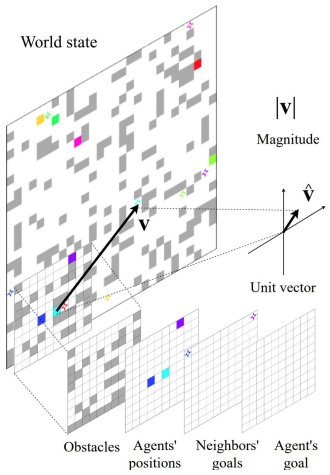
Machine Learning

- PRIMAL: mix of deep reinforcement learning and imitation learning



[1] G. Sartoretti et al., "PRIMAL: Pathfinding via Reinforcement and Imitation Multi-Agent Learning," IEEE RAL 4(3), 2019.

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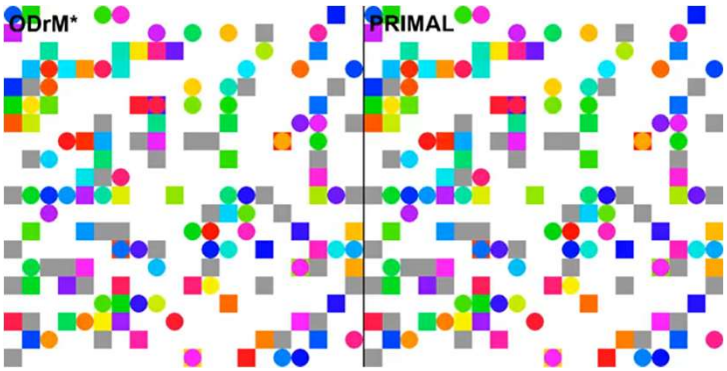
Machine Learning

- Training: 20 days in a supercomputing center
- 64 agents – 20x20 environment – 10% obstacle density



MAPF planner PRIMAL (learning)

[1] G. Sartoretti et al., "PRIMAL: Pathfinding via Reinforcement and Imitation Multi-Agent Learning," IEEE RAL 4(3), 2019.

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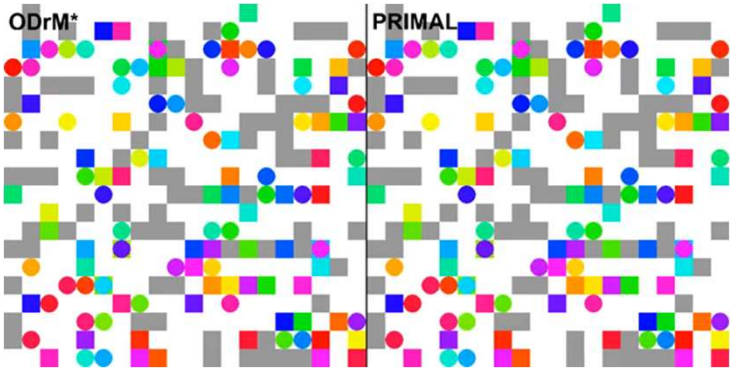
Machine Learning

- Training: 20 days in a supercomputing center
- 64 agents – 20x20 environment – 20% obstacle density



MAPF planner PRIMAL (learning)

[1] G. Sartoretti et al., "PRIMAL: Pathfinding via Reinforcement and Imitation Multi-Agent Learning," IEEE RAL 4(3), 2019.

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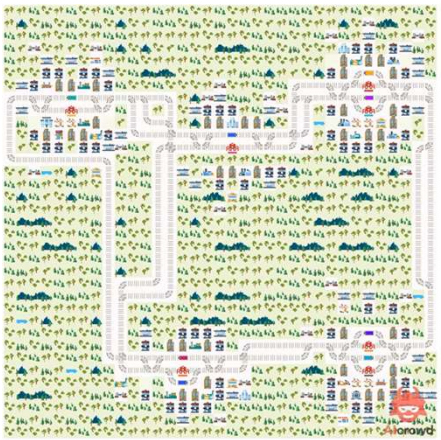
Machine Learning

- NeurIPS 2020 Flatland Competition: 700 participants from 51 countries



MAPF planner

[1] J. Li et al., "Scalable Rail Planning and Replanning: Winning the 2020 Flatland Challenge," ICAPS, 2021.

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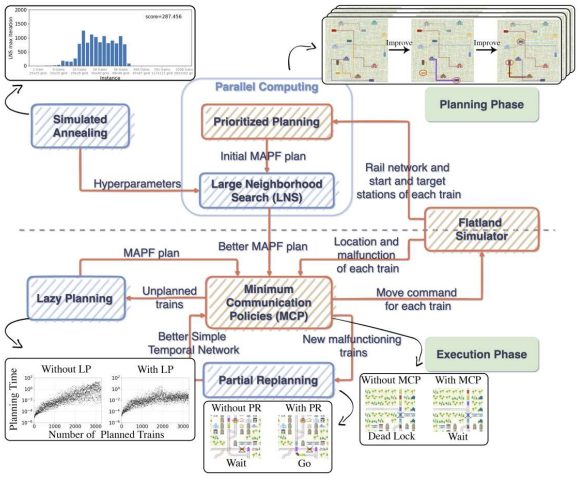
Machine Learning

- NeurIPS 2020 Flatland Competition: 700 participants from 51 countries



[1] J. Li et al., "Scalable Rail Planning and Replanning: Winning the 2020 Flatland Challenge," ICAPS, 2021.

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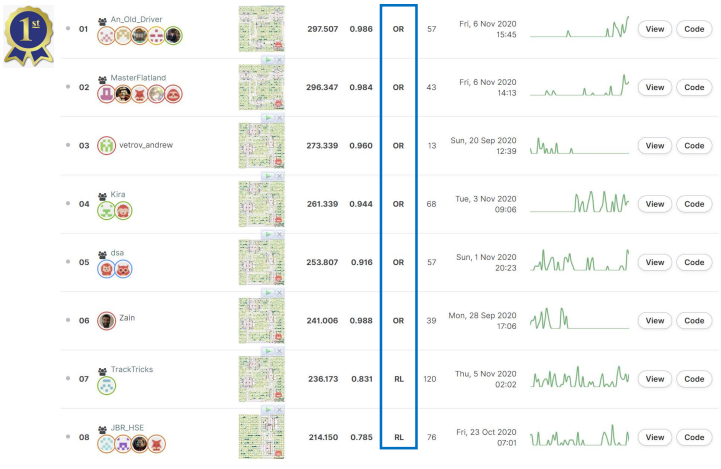
Machine Learning

- NeurIPS 2020 Flatland Competition: 700 participants from 51 countries



[1] J. Li et al., "Scalable Rail Planning and Replanning: Winning the 2020 Flatland Challenge," ICAPS, 2021.

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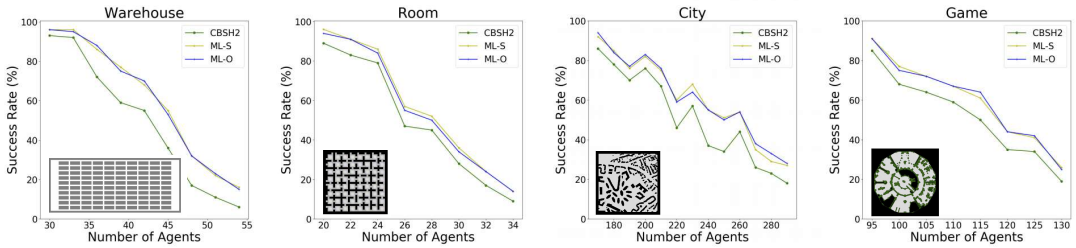
Machine Learning

inspiration from mixed integer programming

- Enhancing planning with machine learning
  - Planners use lots of hard-coded decision strategies
  - Machine learning can often learn to make better decisions
  - The resulting planners can be more efficient and/or effective



[1] T. Huang et al., "Learning to Resolve Conflicts for Multi-Agent Path Finding with Conflict-Based Search," AAAI, 2021.
[2] T. Huang et al., "Learning Node-Selection Strategies in Bounded-Suboptimal Conflict-Based Search for Multi-Agent Path Finding

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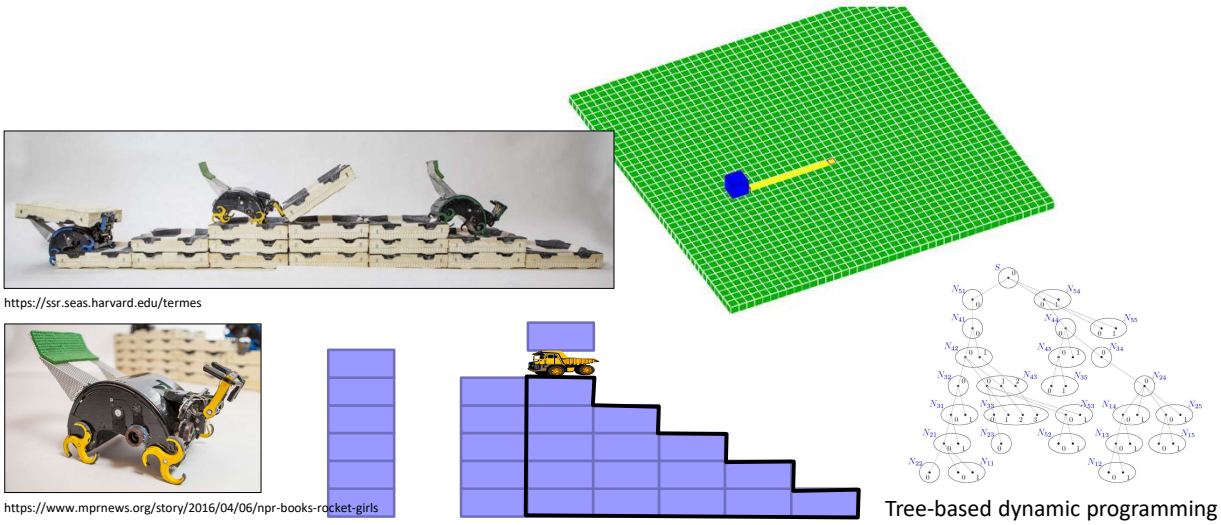
Harvard TERMES Robots



https://ssr.seas.harvard.edu/termes

https://www.mprnews.org/story/2016/04/06/npr-books-rocket-girls

Tree-based dynamic programming

[1] N. Petersen et al., "Termes: An Autonomous Robotic System for Three-Dimensional Collective Construction", RSS, 2011
[2] S. Kumar et al., "A Tree-Based Algorithm for Construction Robots", ICAPS, 2014.

147

Multi-Arm Assembly

[1] J. Chen et al., “Cooperative Task and Motion Planning for Multi-Arm Assembly Systems”, under review for IEEE Robotics a

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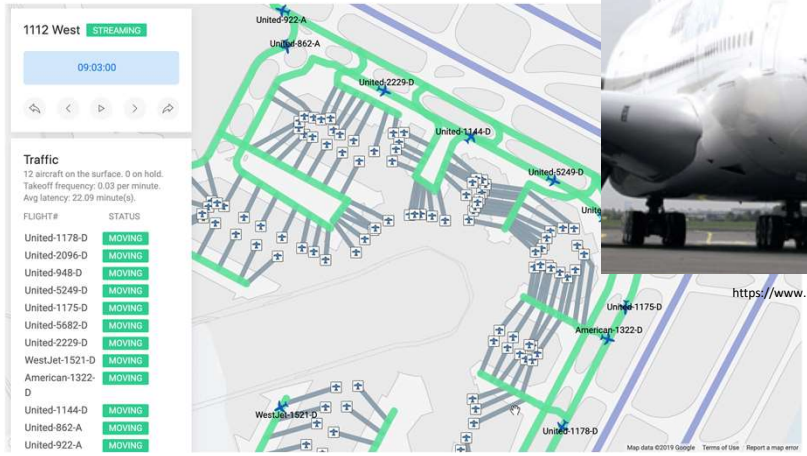
Airport Surface Operation



https://www.kessler-axles.de/en/markets/aircraft-ground-support/aircraft-towing-tractors/

[1] J. Li et al., “Scheduling and Airport Taxiway Path Planning under Uncertainty”, AIAA, 2019.
[2] R. Morris et al., “Planning, Scheduling and Monitoring for Airport Surface Operations”, AAAI-16 Workshop, 2016.

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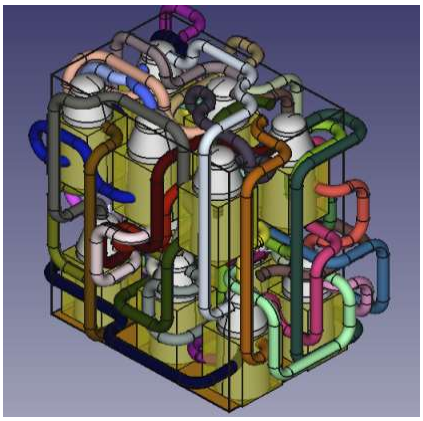
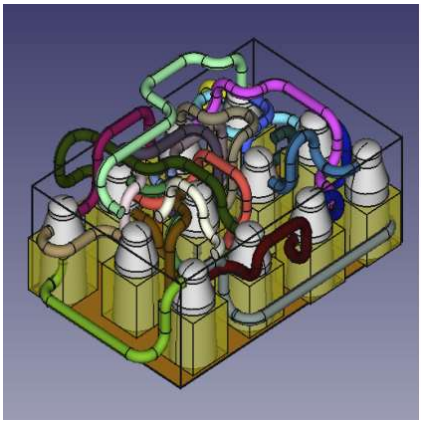
Pipe Routing





x-y-t

↓

x-y-z

[1] G. Belov et al., "From Multi-Agent Pathfinding to 3D Pipe Routing", SoCS, 2020.

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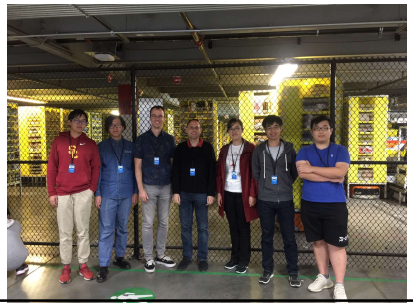
Multi-Agent Path Finding (MAPF)

- H. Andreasson, D. Atzmon, N. Ayanian, R. Bartak, G. Belov, E. Boyarski, V. Bulitko, T. Cai, D. Chan, S.-H. Chan, Z. Chen, H. Choset, M. Cirillo, L. Cohen, B. Dilkina, W. Du, J. Durham, A. Felner, G. Gange, M. Garcia de la Banda, M. Gong, M. Greco, D. Harabor, C. Hernandez, W. Hoenig, T. Huang, S. Jung, J. Kerr, S. Kiesel, S. Kumar, E. Lam, P. Le Bodic, J. Li, Z. Liang, M. Liu, W. Liu, Z. Liu, H. Ma, R. Morris, W. Paivine, C. Pasareanu, F. Pecora, W. Ruml, G. Sartoretti, G. Sharon, Y. Shi, D. Sigurdson, R. Stern, P. Stuckey, N. Sturtevant, K. Sun, P. Surynek, L. Terr, A. Tinka, Z. Tong, C. Tovey, T. Uras, G. Wagner, T. Walker, J. Wang, X. Wei, Y. Wu, H. Xu, M. Yao, W. Yeoh, L. Yi, D. Zhang, H. Zhang, Y. Zheng
- We thank NSF, Amazon Robotics, and NASA Ames for funding several of our projects discussed in this talk!



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Multi-Agent Path Finding (MAPF)

mapf.info webmaster: Sven Koenig

Learn all about Multi-Agent Path Finding (MAPF)

Home Page

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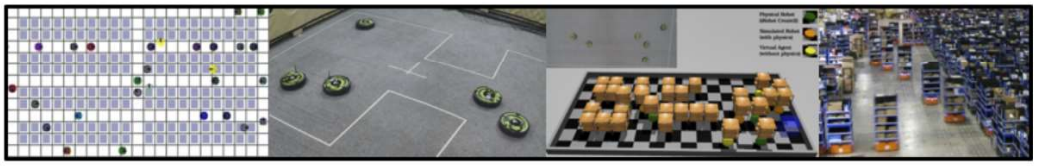
Publications
Researchers
Software
Tutorials
Class Projects

Welcome!

News: MAPF Team wins the NeurIPS-20 Flatland Competition
(click here for more information)



Multi-Agent Path Finding (MAPF) is the problem of computing collision-free paths for a team of agents from their current locations to given destinations. Application examples include autonomous aircraft towing vehicles, automated warehouse systems, office robots, and game characters in video games. Practical systems must find high-quality collision-free paths for such agents quickly.

Consider, for example, automated warehouses. Path planning for robots in such warehouses is tricky since most warehouse space is used for storage, resulting in narrow corridors where robots cannot pass each other. Warehouse robots operate all day long but a simplified one-shot version of the path-planning problem is the simplest form of a MAPF problem, which can be described as follows: On math paper,

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